

Seat No. : _____

DM-122

December-2017

B.C.A., Sem.-I

CC-104 Fundamental Mathematical Concept (FMC) (New)

Time : 3 Hours]

[Max. Marks : 70

1. (A) Let $U = \{x \in \mathbb{Z} \mid 1 \leq x \leq 10\}$, $A = \{x \in \mathbb{Z} \mid 2 < x \leq 6\}$, $B = \{x \in \mathbb{N} \mid 1 \leq x^2 \leq 10\}$.
For these sets answer the following questions : **8**

- (i) Show that $(A \cup B)' = A' \cap B'$.
- (ii) Let a function $f : A \rightarrow U$ defined by $f(x) = x + 1$, then find R_f (Range of f).
- (iii) Find $\{(x, y) \mid x \in A, y \in B \text{ and } x = y\} \subset A \times B$.
- (iv) Show that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

OR

Let $A = \{1, 2, 3\}$ and a function $f : A \rightarrow \mathbb{R}$ defined by $f(x) = \frac{1-x}{1+x}$ then answer the following questions : **8**

- (i) Find $A \cap R_f$ (Range of f)
 - (ii) Find power set of R_f
 - (iii) Show that $f(x) - f\left(\frac{1}{x}\right) = 2f(x)$.
 - (iv) Find $A - R_f$ and $R_f - A$.
- (B) Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \frac{x+1}{2}$, $g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = 2x - 1$, and $h : \mathbb{R} \rightarrow \mathbb{R}$, $h(x) = 3x + k$.
Answer the following questions using above information : **6**

- (i) Find $(f \circ g)(2)$ and $(g \circ f)(0)$.
- (ii) If $h \circ g = g \circ h$ then find k .
- (iii) Show that f is a one-one function.

OR

Let $A = \{1, 2, 5, 6, 7\}$ and $B = \{2, 4, 7, 8\}$ then,

6

- (i) Find $A \Delta B$ and $B \Delta A$.
- (ii) Let $f: A \Delta B \rightarrow B \Delta A$, defined by $f(x) = x$, then show that f is onto function.
- (iii) Let $f: A \rightarrow B \cup \{3, 6\}$ defined by $f(x) = x + 1$. Is f a one-one function? Is f an onto function?

2. (A) Two matrices A and B are given as below

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$$A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 3 & 1 \\ 3 & 2 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 & -2 \\ 4 & -6 & 4 \\ 3 & 1 & -2 \end{bmatrix}$$

Answer the following questions using above matrices :

- (i) Find $|A|$ and $|B|$.
- (ii) Find A^{-1} and B^{-1} if they exist.
- (iii) Verify that $|A + B| = |A| + |B|$.
- (iv) Find the rank of a matrix B .

OR

Using matrix inversion method find the solution of the following system of linear equations :

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$$x + 2y + 3z = 6$$

$$2x + 4y + z = 7$$

$$3x + 2y + 9z = 14$$

Also verify it using Cramer's Rule.

(B) From the given matrix A , find its inverse A^{-1} and also show that $AA^{-1} = I$.

6

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

OR

Let a matrix M of order four :

6

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 4 & 2 \\ 2 & 2 & 2 & 2 \end{bmatrix}$$

- (i) Using properties of determinant, find the rank of a matrix M .
- (ii) Show a matrix M as a sum of symmetric and skew-symmetric matrices.

3. (A) Let four points A(3, 2), B(5, 4), C(3, 6) and D(1, 4) on a XY-plane. 8
- Show that points A, B, C and D are vertices of a square.
 - Using distance formula show that points A, B and C are not collinear.
 - Find the equation of a line passing through A and B.
 - Show that lines \overrightarrow{AB} and \overrightarrow{CD} lines are parallel.

OR

Let two lines $l_1 : x - 2y + 3 = 0$ and $l_2 : 2x - 3y + 4 = 0$. 8

- Find the equation of a line passing through the intersection of l_1 and l_2 and having slope $\frac{2}{3}$.
 - Find the equation of a line passing through the intersection of l_1 and l_2 and parallel to a line joining (1, 1) and (0, -1).
- (B) For given three points A(2, 2), B(7, 8) and C(0, 0) on XY-plane. 6
- Find the ratio in which the line joining A and B is divisible by X-axis.
 - Find the area of a triangle $\triangle ABC$.

OR

Find the equations of two lines passing through (2, -1) and making angle 45° with the line $6x + 5y - 1 = 0$. Also show that these two lines are perpendicular to each other. 6

4. (A) Answer the following questions : 8
- Find $\lim_{x \rightarrow 0} \frac{3^x - 1}{5^x - 1}$
 - Discuss the continuity of $f(x) = [x]$ at $x = 3$.
 - Find $\frac{dy}{dx}$ if $y = x^3 e^{x^4}$
 - Evaluate : $\int x^3 e^{x^4} dx$

OR

Answer the following questions : 8

- Find $\lim_{x \rightarrow 1} \frac{\sqrt{x+2} - \sqrt{3}}{x-1}$
- Let $f(x) = \begin{cases} \frac{x^2 - 36}{x - 3}, & \text{if } x \neq 6 \\ k, & \text{if } x = 6 \end{cases}$. If f is continuous at $x = 6$, find the value of k .
- Find $\frac{dy}{dx}$ if $y = e^{2x+3}$
- Evaluate : $\int \sin(2x + 3) dx$

(B) Answer the following questions :

6

(i) Find $\frac{dy}{dx}$ if $y = x^x$

(ii) Evaluate : $\int_{-1}^1 (3x^2 + 2x + 1)dx$

OR

Answer the following questions :

6

(i) Find $\frac{d^2y}{dx^2}$ if $y = x^3$

(ii) Evaluate : $\int_0^{\pi/2} \sin x \, dx$

5. Do as directed :

14

(1) A relation $f : N \rightarrow N$, $f(x) = x - 1$ is a function. (True / False)

(2) For any non-empty sets A and B $(A \cup B)' = A' \cup B'$. (True / False)

(3) If $A \subset B$ and $B \subset A$ then $A = B$. (True / False)

(4) For any matrix A, $AA^{-1} = I$. (True / False)

(5) A matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is non-singular. (True / False)

(6) Rank of a matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is 2. (True / False)

(7) Two lines $x + y + 1 = 0$ and $x - y + 1 = 0$ are perpendicular. (True / False)

(8) $\int \sin x = \cos x + c$. (True / False)

(9) Let A be a non-empty set with three elements, then $n[p(A)] = \underline{\hspace{2cm}}$.

(10) The range of a function $f : N \rightarrow N$, $f(x) = x$ is $\underline{\hspace{2cm}}$.

(11) If a matrix $M = \begin{bmatrix} 2 & 2x \\ x+1 & 3 \end{bmatrix}$ is a symmetric matrix, then $x = \underline{\hspace{2cm}}$.

(12) The slope of a line $x + y + 1 = 0$ is $\underline{\hspace{2cm}}$.

(13) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \underline{\hspace{2cm}}$.

(14) For $y = e^x$, $\frac{d^2y}{dx^2} = \underline{\hspace{2cm}}$.

DM-122

December-2017

B.C.A., Sem.-I

CC-104 : Basic of Mathematics (BM) (Old)

Time : 3 Hours]

[Max. Marks : 70

1. (A) Let $U = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$, $A = \{1, 2, 3, 4, 5\}$ and $B = \{-5, -3, -1, 2, 4\}$ then find, 8
- (i) $(A \cup B)'$
- (ii) $(A - B) \cup B'$
- (iii) $A' \Delta B'$
- (iv) $(A' - B')'$

OR

Let $f(x) = \frac{x}{x+1}$, then find $f\left(\frac{2}{3}\right)$, $f\left(\frac{1}{x}\right)$, $\frac{f(4) - f(2)}{f(3)}$ and $f(x+1) - f(x-1)$. 8

- (B) From the given Figure – 1 answer the following questions : 6

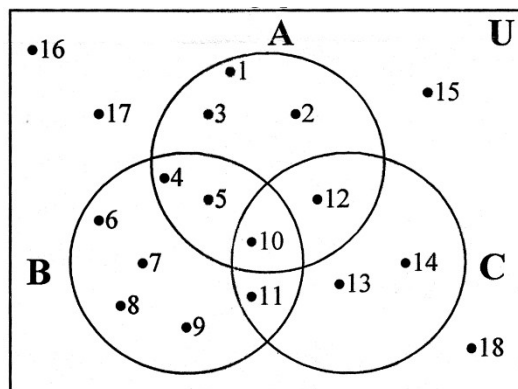


Fig. – 1

- (a) Find $n[(A \cup B)']$
- (b) Find $n(A - C)$
- (c) Find $n(A \times B)$

Where $n(S)$ denotes the number of elements in a set S .

OR

If $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \frac{x+1}{2}$, $g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = 2x - 1$ and $h : \mathbb{R} \rightarrow \mathbb{R}$, $h(x) = x^2$ then prove that $ho(gof) = (hog)of$. 6

2. (A) For given matrices $A = \begin{bmatrix} 2 & 2 & 3 \\ -3 & 2 & 0 \\ -1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix}$ find 8

(a) $A^2 + 2AB + B^2$

(b) $(A + AB)^T$

OR

Express the matrix $A = \begin{bmatrix} 4 & 3 & -2 \\ 3 & 2 & 1 \\ 1 & -4 & -2 \end{bmatrix}$ as a sum of a symmetric and a skew-symmetric matrix. Also verify that $A (\text{adj } A) = |A| I = (\text{adj } A) A$. 8

(B) Solve the following system using Cramer's Rule : 6

$$x + 2y + 3z = 6$$

$$2x + 4y + z = 7$$

$$3x + 2y + 9z = 14$$

OR

Find the rank of matrices, $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$ 6

3. (A) Find the equation of two lines through the point $(2, -1)$ making an angle of 45° with the line $6x + 5y - 1 = 0$. 8

OR

Find the equation of a line passing through the intersection of the lines $x - 2y - 2 = 0$ and $2x - 5y + 1 = 0$ and 8

(a) having Slope $-\frac{1}{2}$

(b) is perpendicular to $3x - 2y + 11 = 0$

(c) is parallel to $2x - 5y + 13 = 0$

(d) having x -intercept is 2

- (B) Show that the points (6, 6), (2, 3) and (4, 7) are the vertices of a right angled triangle. 6

OR

Find the area of a triangle formed by three points (1, 2), (2, 4) and (5, 2). 6

4. (A) (a) $\lim_{x \rightarrow 0} \left(1 + \frac{4x}{5}\right)^{\frac{3}{x}}$

(b) Find $\frac{dy}{dx}$ when $y = 6x^3 - \log x$ 8

OR

(a) Check the continuity of $f(x)$ at $x = 5$. 8

$$f(x) = \frac{x^2 - 16}{x - 4}, x < 4$$

$$= 2x - 4, x \geq 4$$

(b) Evaluate : $\int \frac{1}{4x - 7} dx$

(B) (a) Find $\frac{dy}{dx}$ when $y = x^4 3^x e^x$ 6

(b) Evaluate : $\int \left(\sin t + 3t + \frac{1}{t^2} \right) dt$

OR

(a) Find $\frac{dy}{dx}$ when $y = 7x^3 \log x$ 6

(b) Evaluate : $\int_0^1 (x^2 + 5) dx$

5. Do as directed : 14

- (1) A relation $f : \mathbb{N} \rightarrow \mathbb{N}$, $f(x) = x - 1$ is a function. (True / False)
- (2) For any non-empty sets A and B $(A \cup B)' = A' \cup B'$. (True / False)
- (3) If $A \subset B$ and $B \subset A$ then $A = B$. (True / False)

- (4) List the element of the set $A = \{x / x^3 - x = 0, x \in \mathbb{Z}\}$.
- (5) Give the Range for the function $f : \mathbb{N} \rightarrow \mathbb{N}, f(x) = x + 1$.
- (6) Rank of a matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is 2. (True / False)
- (7) If $A = I_2$ (Identity Matrix of order 2), then find A^2 .
- (8) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \underline{\hspace{2cm}}$.
- (9) Give an equation of a line passing through (4, 0) and parallel to y-axis.
- (10) Give an equation of a line passing through a point $(-3, -2)$ and having slope 2.
- (11) The slope of a line $x + y + 1 = 0$ is $\underline{\hspace{2cm}}$.
- (12) Is the function $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x + 1$ continuous at $x = 0$?
- (13) For $y = e^x, \frac{d^2y}{dx^2} = \underline{\hspace{2cm}}$.
- (14) $\int \sin x = \cos x + c$. (True / False)

$\underline{\hspace{2cm}}$